

UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE

Department of Electrical Engineering

Experiment No. _____ The Analog Computer and Magnitude-Scaling

INTRODUCTION

Often the magnitude of system parameters are such that a system cannot be simulated on an analog computer without exceeding the voltage limits of the computer. In other cases, the parameter magnitudes may be so small that accurate, measurable results are not possible. In such cases, it becomes necessary to magnitude-scale the system parameters in order to simulate the system on the computer and obtain reasonable results.

Consider the stem below which is to be simulated on a +/- 10 volt machine.

$$X''(t) + aX'(t) + bX(t) = cY(t)$$

where,

$$X'_{\max} = 1, \quad X'(0+) = 2$$

$$X_{\max} = 25 \quad X(0+) = -8.$$

In this case, the parameter, $X(t)$, must be magnitude-scaled since its maximum value exceeds the voltage capability of the computer. Also, its initial condition must be magnitude-scaled by the same amount. The magnitude-scaled parameter must be less than 10 volts and should be greater than 0.1 volts. Scaling by a factors of ten is most desirable since the amplifiers generally have a x10 input.

In order not to change the original equation, each parameter must be divided and multiplied by the same scaling factor. Such as,

$$X''(t) + aX'(t) + 10b\left[\frac{X(t)}{10}\right] = cY(t)$$

and,

$$X'_{\max} = 1 \quad X'(0+) = 2$$

$$10 [X_{\max}/10] = 25 \quad \text{or} \quad [X_{\max}/10] = 2.5$$

$$10 [X(0+)/10] = -8 \quad \text{or} \quad [X(0+)/10] = -0.8.$$

One of the outputs of the computer is now $[X(t)/10]$. If this parameter were recorded on a piece of graph paper as a function of time, it would be necessary to scale the output by a factor of ten in order to have a plot of the original parameter as a function of time, or,

$$X(t) = 10 [X(t)/10].$$

It should be pointed out that if the initial condition of a parameter is larger than its maximum value, then the magnitude-scaling is based on the initial condition rather than the maximum value of the parameter.

Often the maximum values of system parameters are not known. In such cases, the problem can be run on the computer while simultaneously magnitude-scaling until satisfactory outputs are obtained.

The problem below not only illustrates magnitude-scaling but also the variety of systems that can be simulated on the analog computer.

The Problem: In the attached figure is an open-top tank which contains a liquid. The liquid leaves the tank at the bottom and is replenished at the top. In order to hold the level of the liquid in the tank at some desired level, H , a controller is used to operate an inlet valve which varies the rate of flow of liquid in the tank. The output flow-rate is determined by the height of liquid in the tank and the viscous drag of the output pipe.

If laminar flow is assumed in the output pipe, then the differential equations describing the height and volume-flow-rate are,

$$A \frac{d^2 h(t)}{dt^2} + \frac{1}{D} \frac{dh(t)}{dt} = \frac{df_1(t)}{dt}$$
$$\frac{df_1(t)}{dt} = K[H - h(t)]$$

where,

$h(t)$ - height of the liquid in the tank
 $f_1(t)$ - volume-flow-rate of the liquid into the tank
 A - cross-sectional area of the tank
 D - coefficient of viscous drag
 K - controller gain
 H - desired height of the liquid in the tank

PRELIMINARY

P-1. Set up equations (1) and (2) above for analog simulation assuming the following parameter values.

$A = 5$ sq. meters
 $H = 15$ meters
 $K = 0.25$ (sq. meters)/sec²
 $D = 0.5$ sec/(sq. meters)
 $h(0+) = 10$ meters
 $h'(0+) = 0$.

The desired outputs are $h(t)$ and $h'(t)$ as a function of time.

P-2. For the system above, the solution for $h(t)$ can be shown to be;

$$h(t) = 15 - 11.18 \exp(-0.2t) \cos(0.1t - 63.43^\circ)$$

Obtain a simulated plot of $h(t)$ and $h'(t)$ on the digital computer using MATLAB (or similar software).

P-3. Draw the analog computer schematic for the problem above being cognizant of the features of the analog computer to be used.

(INSTRUCTOR'S SIGNATURE _____ DATE _____)

PROCEDURE

F-1. Set up the problem from the preliminary report on the analog computer and plot $[h(t)/M]$ and $dh(t)/dt$ as a function of time on an X-Y recorder ("M" is the magnitude-scaling factor). Be sure to record the scales.

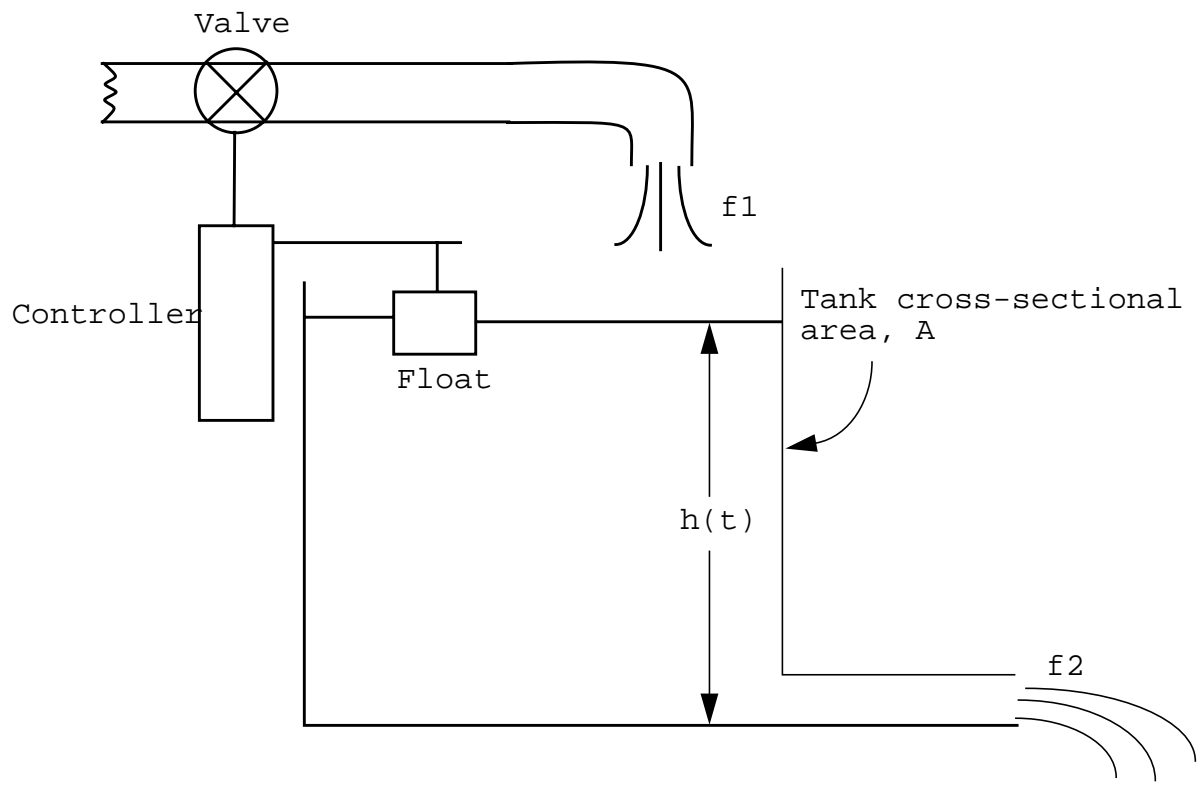
REPORT (should include at least the following)

R-1. Verify the accuracy of your graph using known points (such as $h(0)$ and $h(\infty)$).

R-2. Scale the graph obtained in the procedure to reflect $h(t)$ vs. time.

R-3. Plot the points calculated in the preliminary report on the graphs obtained in the procedure. Discuss any differences.

R-4. Discuss the experiment in terms of magnitude-scaling and of the analog computer as a problem-solving device.



$$A \frac{d^2 h(t)}{dt^2} + \frac{1}{D} \frac{dh(t)}{dt} + Kh(t) = KH$$