X270994

UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE

Department of Electrical Engineering

Experiment No.__________The Analog Computer and Time-Scaling

<u>INTRODUCTION</u>

Often systems to be simulated on an analog computer are too fast to be observed on the output of the computer or to be recorded on a device such as an X-Y recorder. In other cases, the system may have a time response of hours, days, months, or even years. In either case, it may be desirable or even necessary to “time-scale” the system (to slow it down or speed it up).

To “time-scale,” a relationship between real time, tr, and computer time, tc, is needed, such as,

tc = B tr, 1

1

where B is the time scaling constant. Taking the derivative of tc with respect to tr, equation (1) becomes, 1

d(tc)/d(tr) = B. (2)

Consider the system equation,

X”(tr) + a X’(tr) + b X(tr) = c Y(tr). (3)

Using the chain rule,

$$\frac{dX(tr)}{d(tr)} = \frac{dX(tc)}{d(tc)} \times \frac{d(tc)}{d(tr)} = B\frac{dX(tc)}{d(tc)}$$

and,

$$\frac{d^2X(tr)}{d(tr)^2} = \frac{d\left[\frac{dX(tr)}{d(tr)}\right]}{d(tr)} = \frac{d\left[\frac{BdX(tc)}{d(tc)}\right]}{d(tc)} \times \frac{d(tc)}{d(tr)} \qquad (5)$$

$$\frac{d^2X(tr)}{d(tr)^2} = B^2\frac{d^2X(tc)}{d(tc)^2}$$

or,

$$\frac{(tr)}{r)^n} = B^n \frac{d^nX(tc)}{d(tc)^n} \qquad ($$

A time-scaled equation (3) would be,

$$X''(tc) + \frac{aX'(tc)}{B} + \frac{bX(tc)}{B^2} = \frac{cY(tc)}{B^2} \qquad (7)$$

Notice that if a system equation is time-scaled, the input to the system must also be time-scaled.

Time-scaling does not affect the magnitude nor the general shape of a time response curve. To obtain an unscaled time response curve, the time axis is simply re-scaled to real time by dividing computer time, tc, by the scaling constant, B, or,

$$tr = tc/B. \qquad (8)$$

It should also be mentioned that it is desirable that the coefficients of a system function such as that described by equation (3) be between 0.1 and 10 for ease of simulation. Time-scaling along with magnitude-scaling will generally accomplish this.

The problem below not only illustrates time scaling but also the ability of the analog computer to simulate a variety of systems.

<u>The Problem</u>: The discharge of wastes into a body of water presents a problem of primary importance in the field of water-pollution control. The reduction of this organic matter by bacteria results in the utilization of dissolved oxygen. The primary replacement of this dissolved oxygen occurs through the water surface exposed to the atmosphere. An increase in the pollutional load stimulates the growth of bacteria and oxidation proceeds at an accelerated rate. The concentration of the organic load can be so great that all of the dissolved oxygen in a receiving water is utilized by the bacteria. This lack of oxygen inhibits the higher forms of biological life, and conditions set in that are detrimental to man. The concentration of dissolved oxygen is one of the most significant criteria in stream sanitation.

Every stream is limited in its capacity to assimilate organic wastes. As long as this limit is not exceeded, the disposal of organic wastes in streams represents the most economical method of waste disposal. The evaluation of the natural purification capacity of a stream is of fundamental engineering value. Streams are used as natural treatment plants, and it is necessary to determine their capacity in order not to destroy their usage for other purposes.

The simultaneous action of deoxygenation and reaeration produces a pattern in the dissolved-oxygen concentration of river water. This pattern, known as “the dissolved-oxygen sag,” was first described by Streeter and Phelps in 1925. The equation describing the simultaneous action of deoxygenation and reaeration is,

$$dD(t)/dt = K1\ L(t) - K2\ D(t), \tag{9}$$

where,

- D(t) - dissolved oxygen deficit
- L(t) - concentration of the organic matter
- K1 - coefficient of deoxygenation
- K2 - coefficient of reaeration
- D(0+)- initial oxygen deficit at the point of waste discharge.

The concentration of the organic matter, L, can be expressed as follows:

$$dL(t)/dt = - K1\ L(t), \tag{10}$$

where,

- L(0+)- initial concentration of the organic matter in the stream (BOD).

The dissolved oxygen, DO(t), can be determined by subtracting the oxygen deficit, D(t), from the maximum possible oxygen that could be dissolved in the stream for a particular condition,

$$DO(t) = DOmax - D(t). \tag{11}$$

The proportionality factor, K1, is a temperature function. The proportionality factor, K2, is also a temperature function, but more importantly, it is a function of the turbulence of the stream.

PRELIMINARY

P-1. Set up the equations (9) and (10) for the analog computer using the appropriate time scaling.

A city of 200,000 population produces sewage at the rate of 120 gpcd and the sewage plant effluent has a BOD of 28 mg/l The temperature of the sewage is 25.5 degree Celsius, and there is 1.8 mg/l DO in the plant effluent. The stream flow is 250 cfs at 1.2 ft/sec and the average depth is 8 ft. The temperature of the water is 24 degrees Celsius before the sewage is mixed with the stream. The stream is 90 percent saturated with oxygen and has a BOD of 3.6 mg/l. The important coefficients are,

$K1 = 7.03 \times 10^{-6}$ per sec

$K2 = 8.80 \times 10^{-6}$ per sec

D(0+) = 1.59 mg/l

L(0+) = 6.75 mg/l

DOmax = 8.48 mg/l. -

The desired output is DO(t) versus time.

P-2. Draw the analog computer schematic for the problem above being cognizant of the features of the analog computer to be used.

(INSTRUCTOR'S SIGNATURE___________________________DATE__________)

PROCEDURE

F-1. Set up the problem from the preliminary report on the analog computer.

F-2. Plot DO(t) vs. time using an X-Y record. Be sure to record scales.

REPORT (should include at least the following)

R-1. Verify the accuracy of your graph using the digital computer simulated plot..

R-2. Scale the plot obtained from the experiment so that DO(t) is in mg/l and real time.

R-3 Determine the minimum DO of the stream.

R-4. Determine the distance in feet down the stream where minimum DO occurs.

R-5. Discuss the results in terms of time scaling.