UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE

Department of Electrical Engineering

Experiment No.__________Second-Order Systems

INTRODUCTION

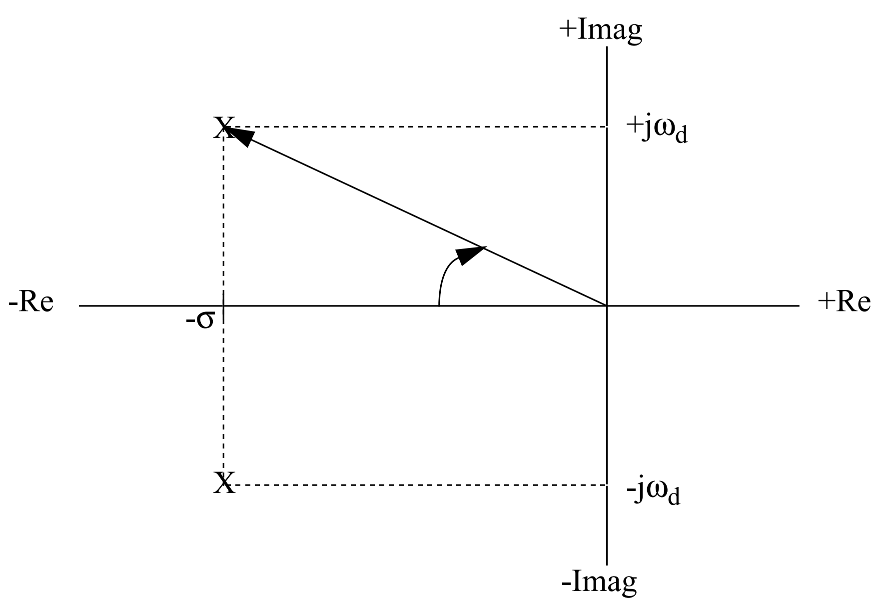
A second-order system can be described mathematically in Laplace form as follows:

$$\frac{X(S)}{Y(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

where ζ is the damping ratio and ω_n is the undamped frequency of the system. The response of the system is dependent upon the roots of the characteristic equation which is the denominator of the above equation. The roots of the characteristic equation are called poles and are expressed as follows for $\zeta <= 1$.

$$S1, S2 = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} = \sigma \pm \omega_d$$

The parameters, σ and ω_d, are called the damping factor and the damped frequency. A second-order pole-plot on the S-plane would look as follows:

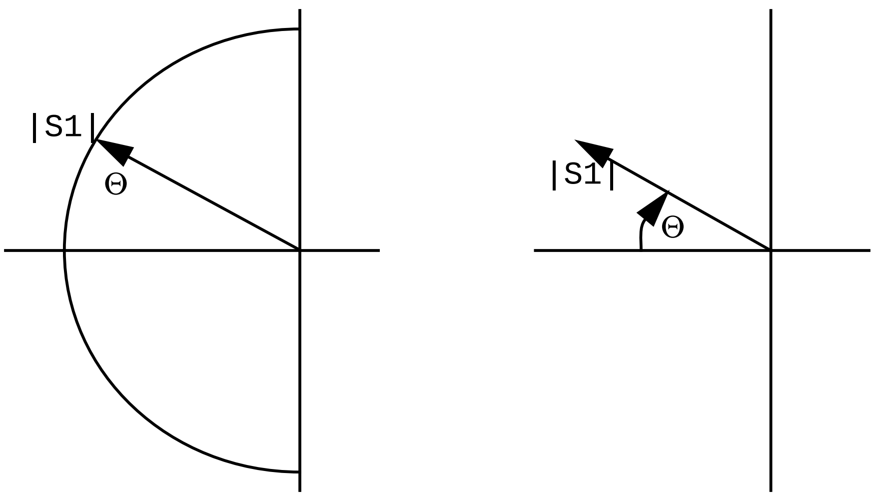


The magnitude of the vector from the origin to the pole location would be,

$$|S1| = \sqrt{(\zeta\omega_n)^2 + (\omega_n\sqrt{1 - \zeta^2})^2} = \omega_n$$

The cosine of the angle from the negative real axis to the vector would be,

$$\cos\Theta = \frac{\zeta\omega_n}{|S1|} = \zeta$$



It can be seen that by holding the magnitude of the vector constant and varying the angle would be in effect holding the undamped natural frequency of the system constant and varying the damping ratio. Also, holding the angle constant and varying the magnitude would be in effect holding the damping ratio constant and varying the undamped natural frequency. It is the purpose of this experiment to vary the damping ratio and undamped natural frequency of a second-order system and determine their effect on the time response of the system. This will be done utilizing the analog computer.

PRELIMINARY

P-1. Writing the above second-order system equation as a function of time with a unit-step input yields:

$$X''(t) + aX'(t) + bX(t) = cU(t)$$

where,

$a = 2\zeta\omega_n$ $\quad$ $b = \omega_n^2$ $\quad$ $c = \omega_n^2$

P-1. (continued)

Draw the schematic for the particular analog computer to be used.

P-2. Make a Table of values for the coefficients a, b, and c for the conditions below.

Case I	ω_n	ζ
#1	3	1.0
#2	3	0.7
#3	3	0.3
#4	3	0.0

Case II	ω_n	ζ
#1	1	0.7
#2	2	0.7
#3	3	0.7

$X'_{max} = 2$, $X_{max} = 2$; $X''(0) = X'(0) = X(0) = 0$.

P-3. Research and write down the definitions and equations for the following terms.

a. Rise time (tr).

b. Delay time (td).

c. Percent overshoot (PO).

d. Settling time (ts).

e. Damped frequency (Wd).

P-4. Obtain simulated plots of Case I, #1-4, and Case II, #1-3, on the digital computer using MATLAB (or similar software).

INSTRUCTOR'S SIGNATURE______________________________DATE_________

PROCEDURE

F-1. Place Case I, condition #1, as specified in the Preliminary exercise, P-1, above on the analog computer and plot the results on a sheet of graph paper using an X-Y recorder. Adjust the time-base and the Y-input of the X-Y recorder so that the response just reaches steady-state at the end of the graph paper and about a third of the way up.

F-2. Repeat F-1 above for Case I, conditions #2, #3, and #4, plotting the results on top of the results of F-1 above. DO NOT RE-ADJUST THE TIME BASE OF THE X-Y RECORDER FROM THAT OF F-1 ABOVE! The Y-input should be adjusted, if necessary, so that all the curves settle out to (or are symmetrical about) the steady-state value of the response of F-1.

F-3. Repeat F-1 and F-2 above for Case II, condition #1, #2, and #3 on a separate piece of graph paper.

REPORT

R-1. Plot the poles for the conditions specified in the Preliminary exercise, P-2, on a single sheet of graph paper.

R-2. Compare the analog computer results with the digital computer results.

R-3. Tabulate the results from P-3, P-4, and from the analog computer for comparison.

R-4. Discuss FULLY the effects of varying ζ and ω_n on the response of a second-order system in terms of rise time (tr), delay time (td), percent overshoot (PO), settling time (ts), and damped frequency (ω_d).