X140994

UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE

Department of Electrical Engineering

Experiment No. _______Resonance in Series and Parallel RLC Networks

INTRODUCTION

An important consideration in the frequency response of a network is the behavior of the network under conditions of resonance. The definitions of some of the more pertinent parameters which describe network resonance are as follows:

1. Resonant frequency (f_r in Hertz and ω_r in radians/second) is the frequency at which the current into the network is in phase with the applied voltage; or, stated another way, the network looks purely resistive.

2. The lower half-power frequency (f_1 or ω_1) and the upper half-power frequency (f_2 or ω_2) are the frequencies below and above the resonant frequency at which the power absorbed by the network falls below 50% of its maximum value. At these frequencies, the magnitude of the current into a voltage-driven, series-resonant network and the magnitude of the voltage across a current-driven,parallel-resonant network are 0.707 of their maximum values. For the voltage-driven, parallel network in this experiment, f_1 and f_2 will be defined simply as the frequencies at which the magnitude of the current is 1.414 of its MINIMUM value.

3. The bandwidth (BW) is the frequency difference between the upper and lower half-power frequencies.

4. The selectivity (S) is the ratio of the resonant frequency to the bandwidth.

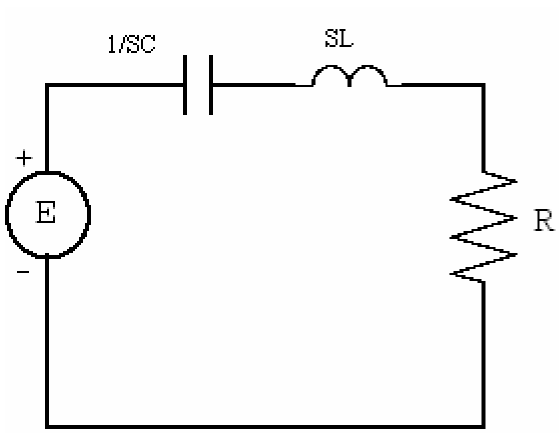
5. The quality factor (Q) is the ratio of the maximum energy stored in the network to the average energy dissipated per radians/second under conditions of resonance.

Using the above basic formulas, closed-form expressions may be derived for the resonant frequency, lower and upper half-power frequencies, bandwidth, selectivity, and quality factor in terms of the network elements for both series and parallel RLC networks.

The purpose of this experiment is to theoretically determine the resonant parameters of a series and a parallel RLC network and compare them with experimental results.

PRELIMINARY

P-1. For the network of Figure 1, derive expressions for fr, f1, f2, BW, and S in terms of R, L, and C.

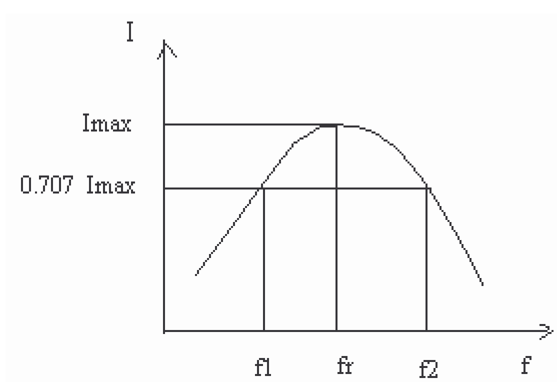


$$I(s) = \frac{E(S)}{R + sL + 1/sC} \qquad s = j\omega$$

$I(j\omega) = E\angle 0 \ / \ R + j(\omega L + 1/\omega C) \ = \ E/R$ at resonance

Hence ω at resonance $\omega_r = 1/\sqrt{LC}$

$\omega_r = 2\pi\, fr$, $\quad fr = 1/\ 2\pi\sqrt{LC}$



$I = E \angle 0 \;/\; \sqrt{R^2+(\omega L - 1/\omega C)^2}$ $\qquad I_{max} = E/R$

$I(f1) = E/R\sqrt{2} = I(f2)$

AT f1 and f2 : $E/R\sqrt{2} = E \;/\; \sqrt{R^2+(\omega L - 1/\omega C)^2}$

$R^2 = (\omega L - 1/\omega C)^2 \qquad \omega^2 L - 1/\omega C = \pm\; R\omega$

$\omega^2 \pm R/L\,\omega - 1/LC = 0$

$\omega = -R/2L \pm \sqrt{(R/L)^2+(4/LC)}$

$\omega 1 = -R/2L + \sqrt{(R/2L)^2+(1/LC)} = 2\pi\, f1$

$\omega 2 = R/2L + \sqrt{(R/2L)^2+(1/LC)} = 2\pi\, f2$

BANDWIDTH = f2 - f1 = R/L

$S = \omega_r \;/\; BW = \omega_r L/R$

P-2. Show that Q = ωrL/R for a series-resonant network.

$$Q = \frac{2\pi\ (\text{MAX ENERGY STORED})}{\text{ENERRGY DISSIPATED PER CYCLE}}$$

$$= \frac{2\pi\ (1/2)\, L\, I^2_{max}}{(I_{max}/\sqrt{2})^2\, R\, Tr} = \frac{2\pi\, fr\, L}{R} = \omega_r L/R$$

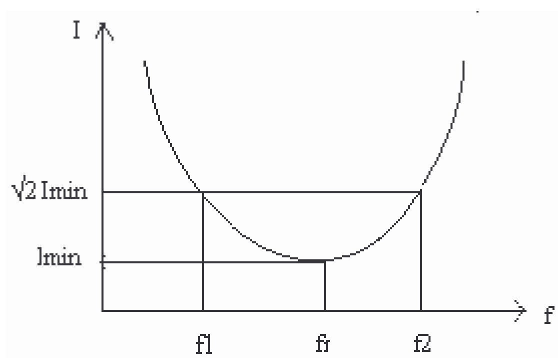
REFER: Pg 656 : Dorf /SVOBODA

P-3. Show the relationship between S and Q for a series-resonant network.

$S = \omega_r \;/\; BW = \omega_r \;/\; \omega 2 - \omega 1 = \omega_r \;/\; (R/L) = \omega_r L/R = Q$

P-4. Assume that C = 0.001 microfarads, L = 300 millihenries, R = 950 ohms (including the resistance of the inductor), and E = 40 volts peak-to-peak. Determine the resonant parameters fr, Q, f1, f2, BW, S, and the magnitude (peak-to-peak) and phase of the currents, Ir, I1, and I2 for the network of Figure 1.

P-5. Derive an expression for the resonant frequency, fr, for the Network of Figure 2. REMEMBER THAT THE PHASE ANGLE BETWEEN THE APPLIED VOLTAGE AND THE INPUT CURRENT IS ZERO AT RESONANCE!



$$Z = \frac{(1/sC)(R+sL)}{R + sL + 1/sC} = \frac{-(j/\omega C)(R+j\omega L)}{R+j\omega L + 1/j\omega C}$$

$$= \frac{L/C - j(R/\omega C)}{R+j(\omega L - 1/\omega C)} = \frac{\text{Num} \angle \Phi 1}{\text{Den} \angle \Phi 2}$$

If $\Phi 1 - \Phi 2 = 0$, then in phase

$$= \frac{\text{num} \angle (-R/\omega C \;/L/C)}{\text{den} \angle (\omega L - 1/\omega C)/R}$$

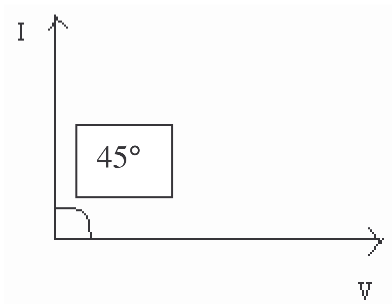
$$-R/\omega L = (\omega L - 1/\omega C)/R \Rightarrow \omega = \sqrt{1/LC - (R/L)^2} \approx 1/\sqrt{LC}$$

P-6.Using the L, C, and R values of P-4, calculate the peak-to peak magnitude and the phase of Ir (fr), I1 (f1), and I2 (f2) for the network of Figure 2.

$$I_{min} = E/R = I_r = 1.266 \text{ X } 10^{-4} \text{ A}$$

$$I(f1) = \sqrt{2}\ I_{min} = 1.79 \times 10^{-4} \angle 45°$$

$$I(f2) = \sqrt{2}\ I_{min} = 1.79 \times 10^{-4} \angle 45°$$



P-7. From the information of P-6, determine the resonant parameters, BW and S, for the parallel-resonant network.

$$S = Q = \frac{2\pi\ [1/2\ C\ V^2 max]}{[(V_{max}/\sqrt{2})^2 RT_r\ /\omega L]} = \omega r\ L / R$$

S = Q for parallel RLC

$$BW = 1/\ R_r C$$

$$S = \omega r\ R_r\ C = R_r/\omega rL$$

P-8. Show that the resistance of a parallel-resonant network at resonance is as shown below.

$$Rr = Q^2 R$$

$$Z = \frac{L/C - j(R/\omega C)\{R - j(\omega L - 1/\omega C)\}}{R + j(\omega L - 1/\omega C)\{R - j(\omega L - 1/\omega C)\}}$$

$$Z_r \approx LR/C(1/R^2) = 1/RC$$

$$Q^2 R = (\omega_r^2 L/R)R = \omega_r^2 L^2/R = (1/LC)L^2/R = L/RC = Z_r$$

(INSTRUCTOR'S SIGNATURE______________________DATE______)

Suggested Reference

Dorf/S.Voboda

PROCEDURE

F-1. Connect the network of Figure 1 using element values of P-4. Using a duel-trace oscilloscope, measure the peak-to-peak magnitude and the phase of the current for frequencies between fr - 2xBW to fr + 2xBW. Maintain the voltage, E, at 40 volts peak-to-peak.

NOTE: MEASURE THE VOLTAGE ACROSS R' AND CONVERT TO CURRENT! ALSO REMEMBER THAT R = R' + RL!

F-2. Connect the network of Figure 2 using elements values of P-4.Make Ri = 500 ohms. Repeat F-1 above except: MEASURE THE VOLTAGE ACROSS Ri AND CONVERT TO CURRENT!

REPORT

R-1. From experimental data, plot the peak-to-peak magnitude and the phase angle of the current for the series-resonant network. Indicate on the graph the frequencies fr, f1, and f2.

R-2. Compare in a Table the calculated and the experimental values for fr, f1, f2, BW, S, Ir, I1, and I2 for the series-resonant network.

R-3. Repeat R-1 and R-2 above for the parallel-resonant network.

R-4. Explain what happens to the resonant parameters of the two networks when the resistance, R, of the networks is increased.

R-5. Repeat R-4 above assuming the resistance, R, is decreased.

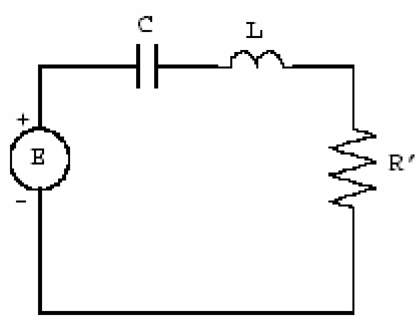


Figure 1

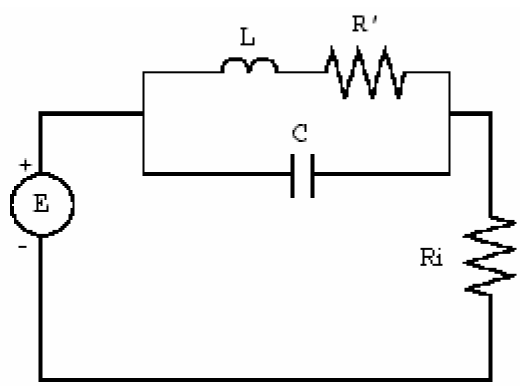


Figure 2