UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE

Department of Electrical Engineering

Experiment No.____________Introduction to the Analog Computer

INTRODUCTION

The operational amplifier (op-amp) is the fundamental component of an analog computer. Internally, an op-amp is very complex. However, it is not essential that one know anything about its internal operation in order to use it, since the external components connected to the terminal of the op-amp determine its function.The symbol for an op-amp used in current textbooks is used here and is shown in Figure l.

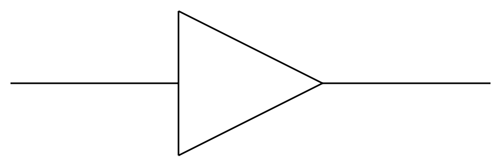

Figure 1. Operational Amplifier

Some important properties of a typical op-amp are:

1. The voltage between the (+) and (-) input terminals is essentially zero.
2. The current drawn by either the (+) or the (-) input terminal is negligible.
3. The amplifier gain is very large (greater than 100,000).

Some of the functions performed by the op-amp that are important to the operation of an analog computer are described below.

THE INVERTING AMPLIFIER: The circuit of Figure 2 is the most widely used circuit involving an op-amp.

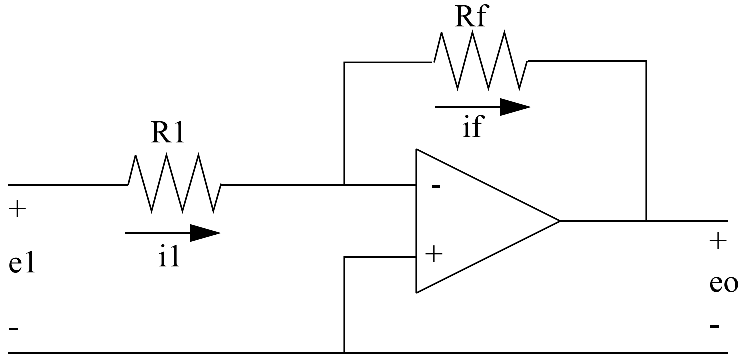

Figure 2. Inverting Amplifier

Applying Kirchhoff's Current law to negative terminal of the amplifier and assuming property 2 above applies;

$$i1(t) = if(t) \tag{1}$$

or,

$$\frac{e1(t) - eg(t)}{R1} = \frac{eg(t) - eo(t)}{Rf} \tag{2}$$

The op-amp's output voltage eo(t) is directly related to the input voltage eg(t) by the amplifier gain, A.

$$eo(t) = -Aeg(t) \tag{3}$$

Substituting equation (3) into equation (2) and rearranging,

$$eo(t) = -\frac{\left(\frac{Rf}{Ri}\right)ei(t)}{\left(\frac{1 + \left(\frac{Rf}{Ri} + 1\right)}{A}\right)} \tag{4}$$

In nearly all applications, A >> (Rf/Ri) + 1, and therefore,

$$eo(t) = -\left(\frac{Rf}{Ri}\right)ei(t) \tag{5}$$

Thus, the input-output relationship is dependent solely upon the ratio of the input and output resistances. In the case where Rf < Ri, the circuit acts as an inverting attenuator. If Rf = Ri, the circuit acts simple as an inverter or sign changer. For Rf > Ri, the circuit acts as an inverting amplifier.

THE SUMMER: In the case where there are "n" number of inputs to the negative terminal of the op-amp such as shown in Figure 3, the output would be the negative sum of all of them.

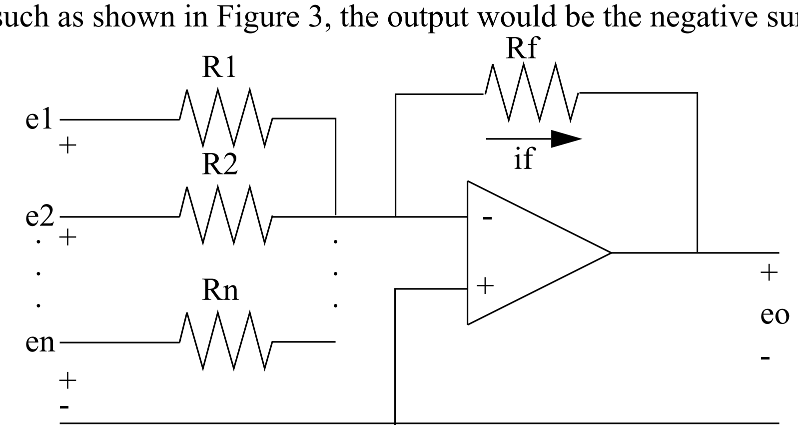

Figure 3. Summing Amplifier

$$eo(t) = -\left(\left(\frac{Rf}{R1}\right)e1(t) + \left(\frac{Rf}{R2}\right)e2(t) + \ldots + \left(\frac{Rf}{Rn}\right)en(t)\right)$$

INTEGRATORS: The op-amp circuit used to perform integration uses a capacitor in the feedback circuit instead of a resistor. The schematic is shown in Figure 4.

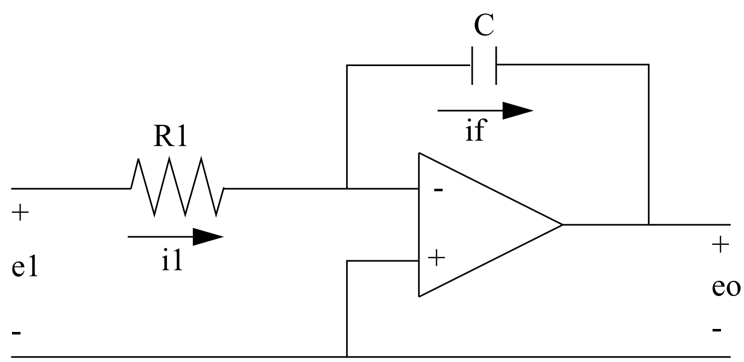

Figure 4. Integrator

In this case,

$$eo(t) = \left(\frac{1}{C}\right)\int_0^t (-if(t))dt + eo(0+) \tag{6}$$

where eo(0+) is the initial value of the voltage on the capacitor.

Since,

$$if(t) = i1(t) = \frac{e1(t)}{R1}$$

then,

$$eo(t) = -\left(\frac{1}{C}\right)\int_0^t e1(t)dt + eo(0+) \tag{7}$$

Summing can also be accomplished while integrating.

$$eo(t) = -\left(\frac{1}{C}\right)\int_0^t \left(\frac{e1(t)}{R1} + \frac{e2(t)}{R2} + \ldots\right)dt + eo(0+) \tag{8}$$

Most analog computers have relays that short out the capacitors whenever the analog computer operation switch is placed on STANDBY. This is to prevent a voltage from building up on the capacitor. All the relays open simultaneously when the analog computer operation switch is placed on RUN. This arrangement also provides a way to place a known initial condition on the capacitor.

MULTIPLICATION BY A CONSTANT OF LESS THAN UNITY: It is often desirable to multiply a variable by a constant of from zero to unity. This can be done very easily using a potentiometer as shown in Figure 5.

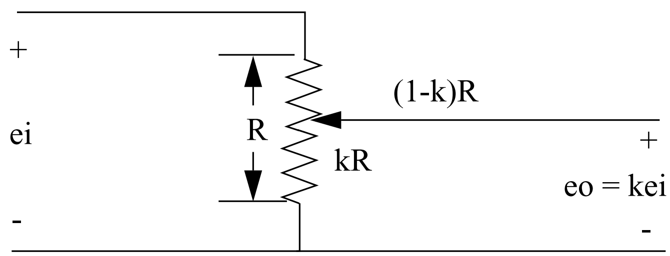

Figure 5. Potentiometer

Using the potentiometer rule,

$$eo = \left(\frac{kR}{R}\right)ei = kei \tag{9}$$

As can be seen in Figure 5, the value of K can vary from zero to one.

If the output of the potentiometer is fed into a circuit that does not have an infinite input resistance, then there will be a "loading effect" that will change the gain of the potentiometer. To insure that potentiometers are set at their correct gain, they are ALWAYS set AFTER the problem to be simulated is completely "patched" onto the analog computer.

ANALOG SIMULATION: Consider the standard form of a second-order, linear, differential equation.

$$\ddot{y}(t) + 2\delta\omega_n\dot{y}(t) + \omega_n^2 y(t) = f(t) \tag{10}$$

The first step in preparing the layout of the computer setup is to rearrange the equation so that the highest derivative appears by itself on the left side of the equation,

$$\ddot{y}(t) = -2\delta\omega_n\dot{y}(t) - \omega_n^2 y(t) + f(t) \tag{11}$$

Next, assume that $\ddot{y}(t)$ is available; then $-\dot{y}(t)$ can be generated at the output of an integrator as shown in Figure 6.

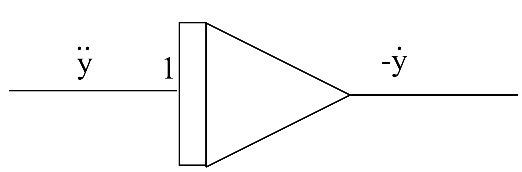

Figure 6. Integrator

Now, by connecting the output of this integrator to the input of a second integrator, it is possible to generate y(t). This is shown in Figure 7.

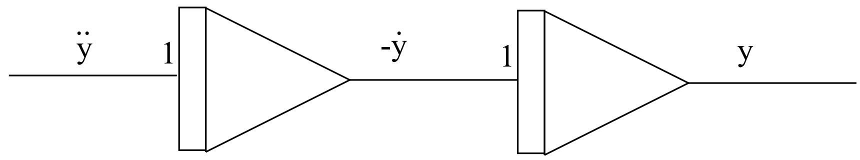

Figure 7.

Referring to equation (11) above, $-\ddot{y}(t)$ can be generated from the output of the two integrators plus an input f(t) connected as shown in Figure 8.

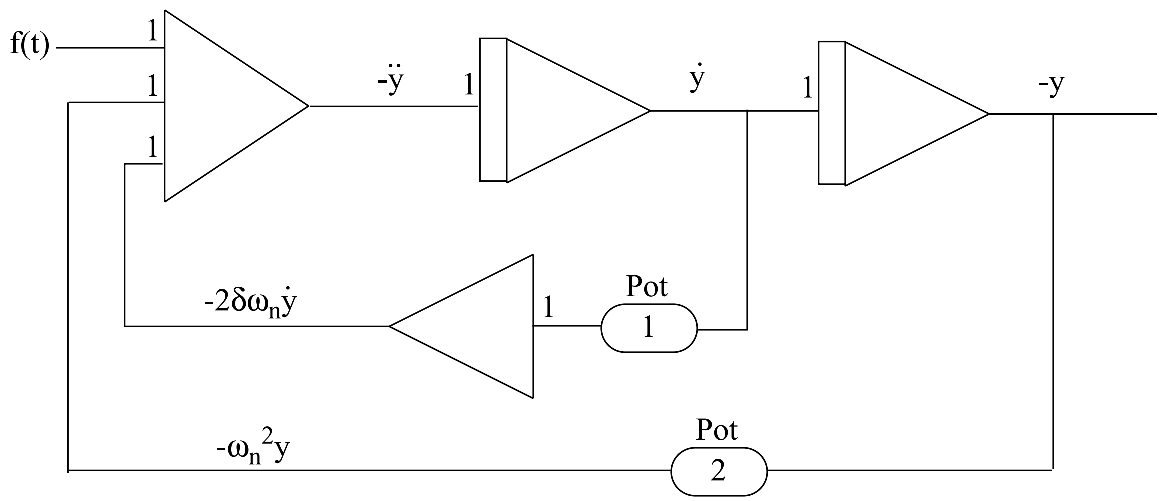

Figure 8.

Potentiometers 1 and 2 are set as $2\delta\omega_n$ and ω_n^2 respectively.

If it is not necessary to observe $\ddot{y}(t)$, then the summer-inverter can be eliminated and the first integrator can be used as the summer. This arrangement is shown in the computer setup of Figure 9.

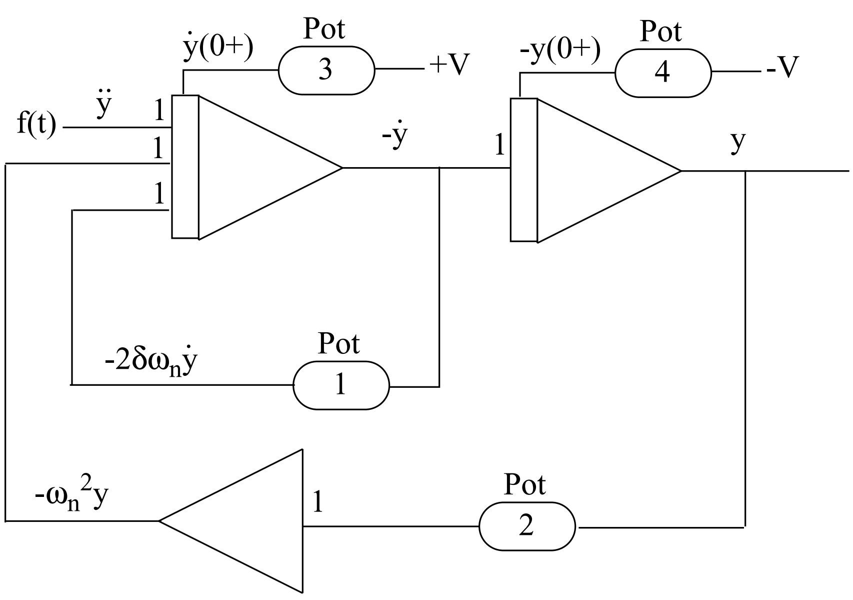

Figure 9.

The differential equation also has initial conditions associated with it. The initial conditions are included in the computer solution by placing initial voltages on the capacitors of the integrators. The input of initial conditions is included in the computer setup of Figure 9. CARE MUST BE EXERCISED when setting initial conditions to insure that the polarity of the initial condition is correct. When the initial condition of the variable is positive but the negative value of the variable appears at the output of an integrator, then the initial condition must be set to produce a negative output on the integrator. The Table below should help in setting up initial conditions.

Polarity of the Initial Condition	Polarity of the Output of the Integrator	Polarity of the Initial Condition "SET" Voltage
Positive	Positive	Negative
Positive	Negative	Positive
Negative	Positive	Positive
Negative	Negative	Negative

The analog computer is used to simulate a mathematical model of a physical system. The analog computer is a physical system itself, where the independent variable is time and the dependent variables are the output voltages of the amplifiers. The problem variables vary in a manner analogous to the computer variables. Since the output voltage range of the amplifiers is limited to the operating range of the analog computer (usually + 10 volts or + 100 volts), it is often necessary to magnitude-scale the computer variables so that the maximum value of the mathematical variable will not cause the computer variable to exceed the operating voltage limits of the analog computer.

It is often desirable to time-scale, to speed up or slow down the solution, depending on the recording device to be used to record the analog solution.

Magnitude- and time-scaling are beyond the scope of this experiment and will be discussed in later experiments.

COMPUTER OPERATION: The operating and patching procedures for the COMDYNA GP-6 analog computer are presented in an attachment to this experiment courtesy of COMDYNA, INC. These procedures are typical of most analog computers of this size.

PRELIMINARY

THE PROBLEM: This experiment is designed to introduce the student to the analog computer through the simulation of a mechanical system that can be represented by a second-order, linear, differential equation. The system is shown in Figure 10 below. The system parameters are as follows:

Mass, m = 0.764 Kg

Spring Constant, k = 0.447 N/M

Damper, b = 0.728, 0.661, 0.468, 0.132, and 0.0

The input is a force, f(t) = 1.0 u(t)

The equilibrium position, xo = 10 meters from the wall

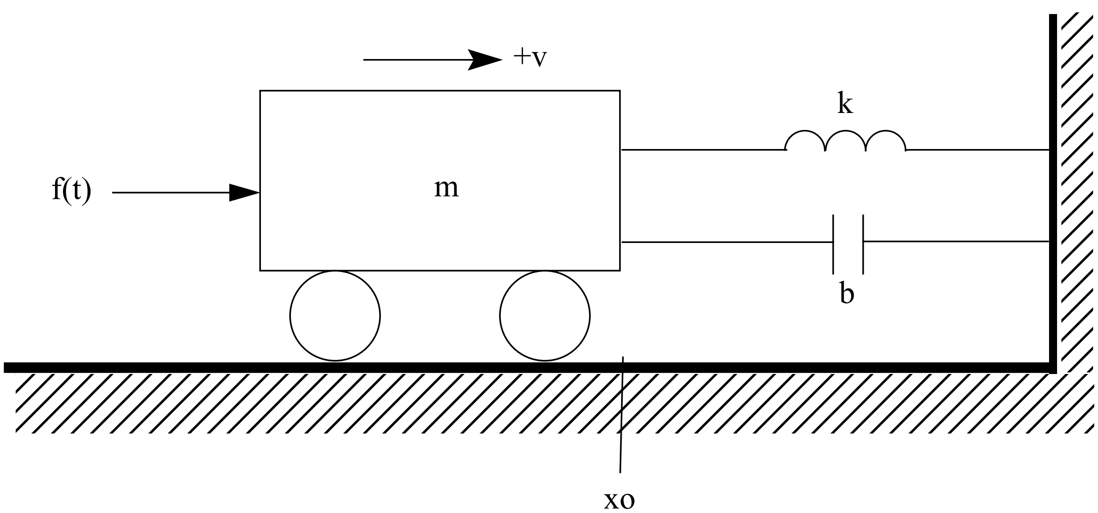

Figure 10. Mechanical System

P-1. Draw an electrical circuit to represent the mechanical system above.

P-2. Formulate the differential equation describing the mechanical system above in terms of the velocity of the mass.

P-3. Formulate the differential equation describing the mechanical system above in terms of the displacement of the mass ($v(t) = dx(t)/dt$). The displacement of the mass will be about the equilibrium position, xo.

P-4. Draw an analog schematic that will simulate the mechanical system above in terms of displacement x(t) but in which the second derivative of x(t) does not appear as an output of an amplifier (see Figure 10). Provide inputs for initial conditions and outputs for both velocity and displacement.

P-5. Determine the maximum values of v(t) and x(t) for a unit-step force input and a damping, b = 0. Use this information to determine the scales needed to produce a reasonable X-Y recording assuming the following scaling factors available on the X-Y recorder:

Time Sweep Settings (mm/sec)	Y-Input Settings (mV/cm & V/cm)
0.05	0.05
0.1	0.1
0.2	0.2
0.5	0.5
1	1
2	2
5	5
10	10
20	
50	
100	

(INSTRUCTOR'S SIGNATURE______________________________________DATE_________)

PROCEDURE

F-1. "Patch" the analog computer according to the schematic in the P-4 above.

F-2. Adjust the Time-sweep and Y-input settings of the X-Y recorder to those determined in P-5 above. Obtain a plot of velocity versus time for each of the five damper settings specified in the Preliminary report; placing all five plots on the same piece of centimeter graph paper.

F-3. Repeat F-2 above for distance versus time.

F-4. Using the same settings as in F-2 above, obtain four plots of distance versus time, all on the same graph paper, for the following initial conditions,

$x(0+) = -9, -3, 0,$ and 3 meters

assuming,

$b = 1.15$ Newton-seconds/meter

$x'(0+) = v(0+) = 0$ meters/second.

F-5. Repeat F-4 above for velocity versus time.

REPORT

R-1. Explain what is physically taking place in the above system when initial displacements are included. Do the plots agree with the physical interpretations?

R-2. Explain the effect of the initial conditions on the steady-state values of distance and velocity.

R-3. With the mass initially 10 meters from the wall (no initial displacement), would the mass hit the wall for $b = 0.132$ N-sec/m?

R-4. Discuss the usefulness of the analog computer as a tool to simulate various systems.

OPERATIONAL PROCEDURES FOR THE COMDYNA GP-6 ANALOG COMPUTER

1. BEFORE OPERATING THE COMDYNA GP-6 ANALOG COMPUTER:

Make sure that the OP OUTPUT and the OP INPUT terminals on the back of the computer are jumpered together. Below is a description of the terminals on the back of the computer.

Terminal	Color	Description
0	Red	-15 volts
1	Green	+15 volts
OP OUTPUT	Red	Integrator mode control logic output.
OP INPUT	Green	Integrator mode control logic input. For normal operation, a jumper wire connects OP OUTPUT to OP INPUT. For slave operation, see Operator’s Manual.
X OUTPUT	Red	Output of the X ADDRESS switch and horizontal input to readout instruments.
GND	Black	Signal ground.
Y OUTPUT	Red	Output of the Y/POT ADDRESS switch (amplifier output section) and vertical input to readout instruments.
METER INPUT	Green	Input to the internal digital voltmeter while in the OPERATE mode.

2. The ON-OFF switch is part of the COMPUTE TIME control.

3. The IC push-button would normally be depressed while setting up and/or patching the analog computer.

The functions of the push-buttons controls are as follows:

IC - Initializes the system integrators (RESET).
HD - Places system integrators into a hold condition (HOLD).
OP - Places system integrators into a normal operating mode (RUN).The MODE SELECTOR switch would normally be in the OP position.
RO - Automatically reduces the time constants of the system integrators by a factor of 400 (FAST) and repetitively switches the system integrators from an IC to an OP condition. The sweep period can be varied with the COMPUTE TIME control.

4. SETTING THE COEFFICIENT POTENTIOMETERS:

Coefficient potentiometers are ALWAYS set AFTER the problem has been COMPLETELY patched on the computer.

If any amplifier is in an OVERLOAD condition (indicated by the OVLD light), the OVERLOAD must be removed before setting the coefficient potentiometers. This can be done by patching the output of the amplifier back to its input SJ.

If potentiometers #7 and #8 are to be used as coefficient potentiometers, their bottom terminals must be patched to ground.

To set the coefficient potentiometers:

a. Position the MODE SELECTOR switch to POT SET.
b. Position the Y/POT ADDRESS switch to the number of the potentiometer to be set.
c. Adjust the selected potentiometer until the desired setting is observed at the internal digital voltmeter (0 to 1).

5. The output of any amplifier can be measured using the ADDRESS switches and the internal digital voltmeter.

To measure the output of an amplifier with the MODE SELECTOR switch in the POT SET position:

a. Place the Y/POT ADDRESS switch in the GND/X position.
b. Position the X ADDRESS switch to the number of the amplifier for which the output is to be measured.
c. The reading of the internal digital voltmeter must be multiplied by 10 to get the actual voltage reading.

To measure the output of an amplifier with the MODE SELECTOR switch in the OP position:

a. Patch the Y OUTPUT terminal on the back of the computer to the METER INPUT terminal on the back of the computer.

b. Position the Y/POT ADDRESS switch to the number of the amplifier for which the output is to be measured.
c. The reading of the internal digital voltmeter must be multiplied by 10 to get the actual voltage reading.

6. External signals can be measured using the internal digital voltmeter by patching the signal into the METER INPUT (and GND) terminals on the back of the computer and placing the MODE SELECTOR switch in the OP position.

7. The input of an integrator (sum) can be measured in the following manner:

   a. Place the MODE SELECTOR switch in the OPR position.
   b. Depress the IC push-button switch.
   c. Patch the SJ terminal of the integrator for which the input is to be measured to an SJ terminal of an unused amplifier.
   d. Measure the output of the amplifier using procedures outlined in Procedure 5 above. The output of the amplifier is the inverted value of the integrator input times the gain of the amplifier.