

X110994

## UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE

Department of Electrical Engineering

Experiment No. \_\_\_\_\_ Time-Constant of an RC Network

The purpose of this experiment is to obtain the time-constant of an RC, series-connected network. There are several methods that could be used to accomplish this. One method would be to obtain a transient response plot of the current in the network or of the voltage across the capacitor in the network and note the time it takes for the measured current or voltage to reach 63.2 percent of its steady-state value. Another method would be to project the INITIAL slope of the transient response plot to the time-axis, and the point at which it intersects the time-axis would be the time-constant. Both of these methods are illustrated in Figures 1 and 2.

Still another method would be to plot the logarithmic values of the current or voltage as a function of time. This will result in a straight line with an intercept and a slope as shown in Figure 3. To illustrate this mathematically, consider the equation for the current in the network of Figure 4:

$$i(t) = i_o \exp(-t/t_c)$$

$$\log(i(t)) = \log(i_o \exp(-t/t_c))$$

$$\log(i(t)) = \log(i_o) + \log(\exp(-t/t_c))$$

$$\log(i(t)) = \log(i_o) - (0.434/t_c) t \quad (1)$$

This would plot as bels (a unit of attenuation) versus time. If the plot were to be in decibels versus time, the above equations would need to be multiplied by 20, i.e.,

$$20 \log(i(t)) = 20 \log(i_o) - (8.68/t_c) t \quad (2)$$

The equations (1) and (2) above are in the form of the equation of a straight line,  $y = b + mx$ , where for equation (2),

$$y = 20 \log(i(t))$$

$$b = 20 \log(i_o)$$

$$m = - 8.68/t_c$$

$$x = t$$

The time-constant,  $t_c$ , can be obtained from the slope of the logarithmic plot.

$$t_c = - 8.686 (.t/.20 \log(i(t)))$$

The change in the time divided by the change in the current in decibels will be negative; thus, making the time-constant positive.

### Useful information

First Order Ckt Response given by:  $dX/dt + aX = 0$

Solution of a first order diff eqn :

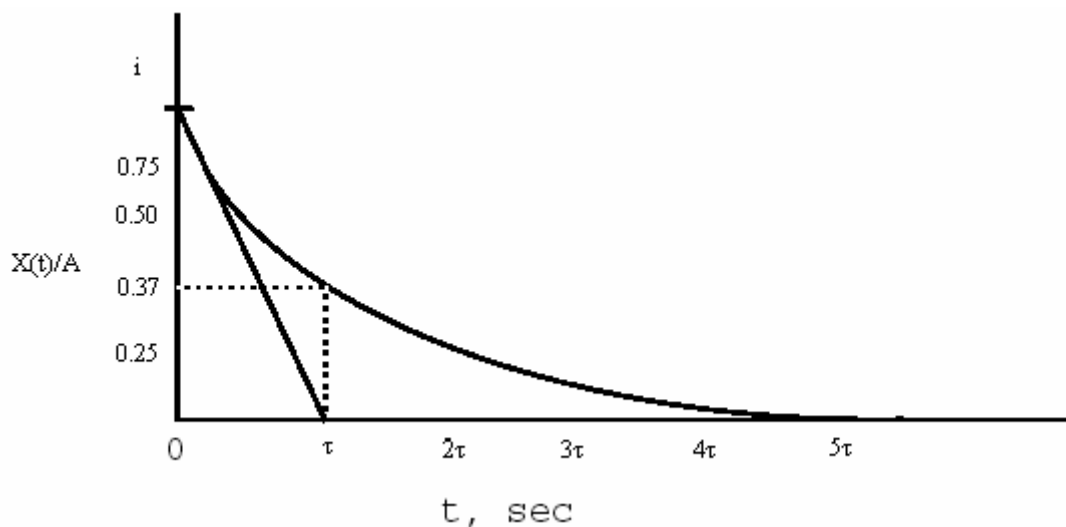
$$X = A e(\exp -at) \text{ or } X = A e(\exp -t/\tau) \text{ where } \tau=1/a$$

For a RL circuit,  $\tau=L/R$

For a RC circuit,  $\tau=RC$

For RC : since the circuit will exhibit an exponentially decaying waveform , an example is shown below.

$t=n\tau$	$\tau$	$2\tau$	$3\tau$	$4\tau$	$5\tau$
$e(\exp -t/\tau)$	0.368	0.135	0.05	0.018	0.007



If an RC circuit is to respond rapidly, a small time-constant is needed. On the other hand, to hold the capacitor voltage near its original value, a large time-constant is needed. Therefore, the design engineer can use the selection of the circuit elements suitably to obtain the desired circuit response.

Capacitor voltage  $V_c = V_o e^{-(t/\tau)}$

When time-constant  $\tau = RC$ , the response declines to 36.8% of its initial value at  $t=\tau$  which can be seen in the graph above.

If we wish to hold the capacitor voltage equal to  $V_o$  for 10 sec, we will need a large RC product. For example, if we wish the voltage decay only 10%, we would require:

$$0.9V_o = V_o e^{-(t/\tau)}$$

$$\text{So } t/\tau = 0.105$$

Since  $t = 10$  sec, we need  $\tau = 95$  which can be achieved by selecting R say 10k and  $C = 9.5$  Mf

#### Suggested Reference

Basic Engineering Circuit Analysis by J.David Irwin  
Fourth Edition (Chapter 7)

Electric Circuits by Dorf/Svoboda

#### PRELIMINARY

P-1. Using circuit equations derive an expression for the current in the circuit of Figure 4 as function of time. Assume the switch, S, is OPENED at  $t = 0$  and the value of the capacitor is unknown (carry C as an unknown in the equation).

$$i(t) = i_o \exp(-t/RC)$$

$$i(t) = i_o \exp[-t/(1.002 \times 10^6)C]$$

P-2. Determine the time-constant,  $t_c$ , from the equation in P-1 above for  $i = 0.368 i_o$  (63.2 percent change in current) and capacitor values of 8, 16, and 24 uF.

$$\ln(0.368) i_o = i_o \ln \exp(-t/1.002 \times 10^6 C)$$

$$t = 0.999 / [(1.002 \times 10^6) C]$$

$$t = 0.1246, 0.0623, 0.04154 \text{ sec (C = 8, 16, 24 } \mu\text{F)}$$

- P-3. Take the derivative of the equation in P-1 above and find the time-constant by setting  $t = 0$  and equating the results to  $i_0/tc$ . Do this for capacitor values of 8, 16, and 24  $\mu\text{F}$ .

$$d i_0 / dt = i_0 d e^{-t/RC} / dt$$

$$0 = e^{-t/RC}$$

$$\ln 0 = \ln e^{-t/RC}$$

$$1 = -t/RC$$

$$t = |RC|$$

(INSTRUCTOR'S SIGNATURE \_\_\_\_\_ DATE \_\_\_\_\_)

### PROCEDURE

- L-1. Turn on the logarithmic converter and let it be warming up.
- L-2. Before making a plot, close and open switch S noting the time that it takes the Y-input to the X-Y recorder to transition to within 10 percent of its final value. Use this time to set the time-sweep of the X-Y recorder.
- L-3. To obtain a plot, make sure switch S is closed and the pen of the X-Y recorder is down and LEFT of the vertical axis ( $t = 0$ ). Start the time-sweep and open the switch S just as the pen crosses the vertical axis.
- L-4. Place the logarithmic converter between the circuit and the Y-input to the X-Y recorder. If the X-Y recorder being used is a LINSEIS Model LY 1800, strap a 20-Kohm resistor across the output terminals of the logarithmic converter (this is necessary in order to obtain an accurate plot using this recorder). A Table showing the output of the logarithmic converter in decibels/millivolt (dB/mv) is shown below.

These can be used to scale plots obtained for various Y-settings of the X-Y recorder.

Obtain a logarithmic plot of current versus voltage.

Logarithmic Converter Output	
(dB/inch)	(dB/mV)
5	0.5
10	1
20	2

R-1. Determine the time-constant,  $t_c$ , from the experimental data using the three methods discussed in the Introduction.

R-2. Compare the three values of the time-constant obtained from experimental data with each other and with values obtained in the Preliminary, P-2 and P-3.

R-3. Discuss the results of the experiment.

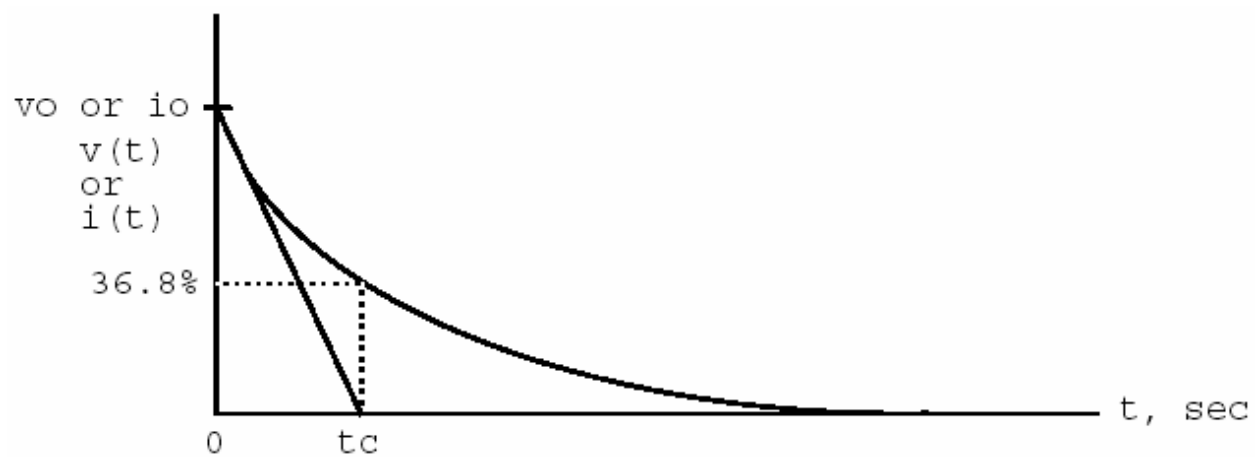


Figure 1.

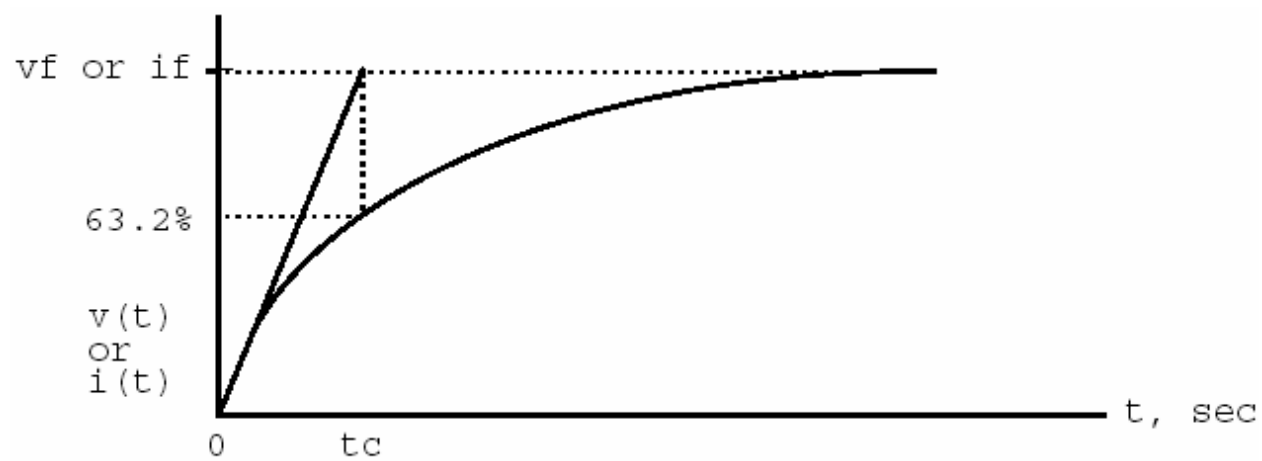


Figure 2.

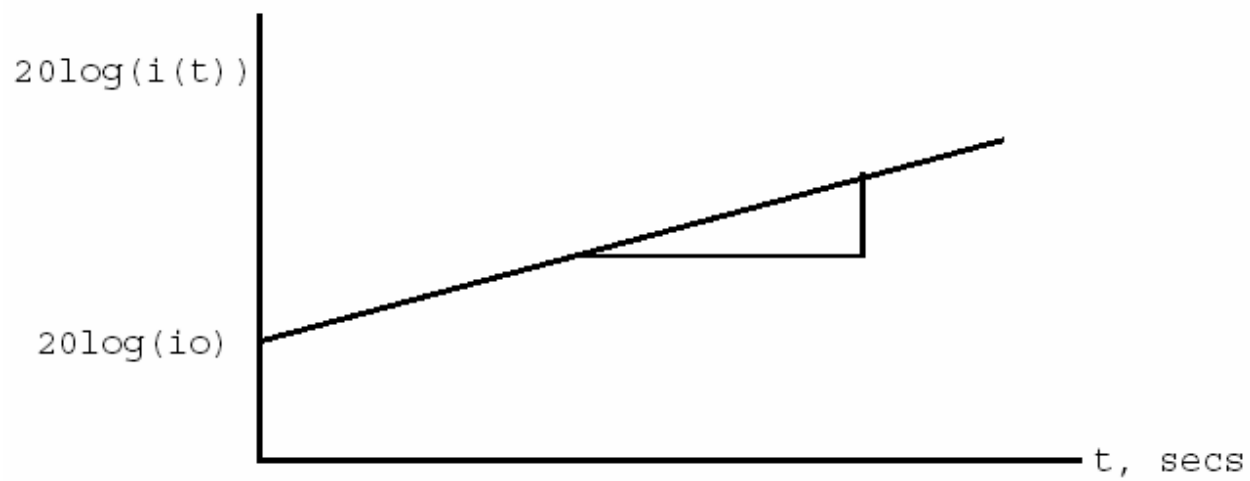


Figure 3.

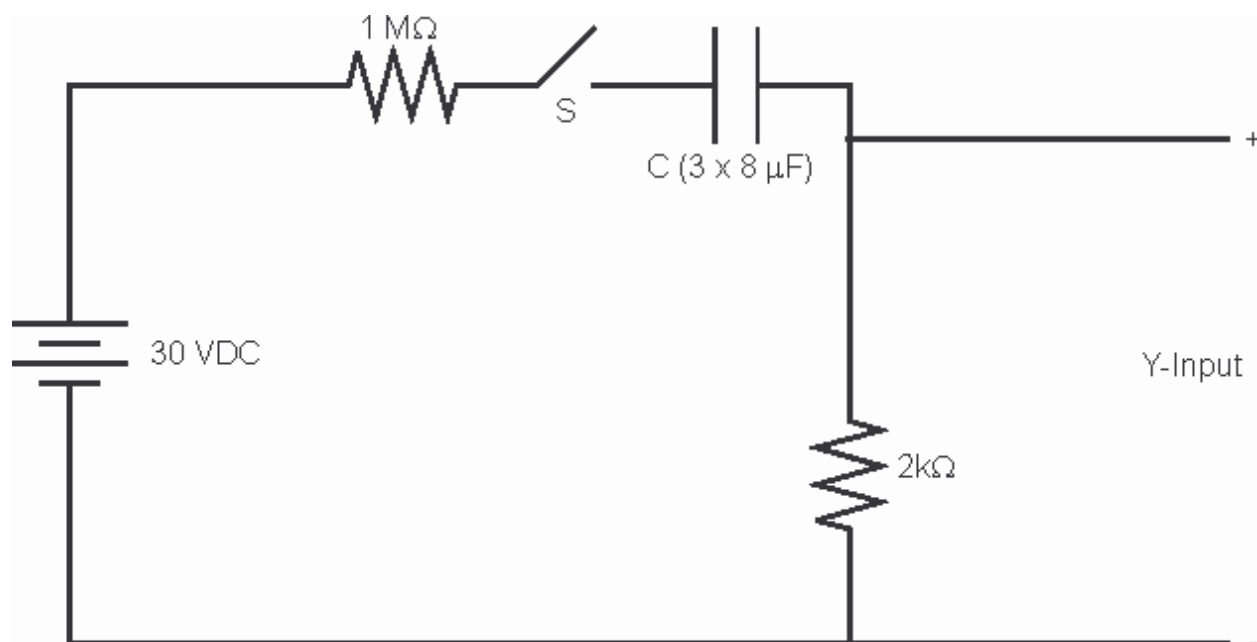


Figure 4